mathematics

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Controllability of Semilinear Systems with Multiple Variable Delays in Control

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Received: 3 September 2020; Accepted: 28 October 2020; Published: 4 November 2020

Abstract: In the paper semilinear, finite-dimensional, control systems with multiple time variable point delays in admissible controls are considered. Using Rothe's fixed-point theorem, sufficient controllability conditions are formulated. The results of the paper are generalization to many time variable delays in control, of the results published recently.

Keywords: controllability; semilinear control systems; delayed control systems; Rothe's fixed-point theorem

1. Introduction

The last decades have seen a continually growing interest in controllability theory of dynamical control systems with different delays [1–5]. This is clearly related to the wide variety of theoretical results and possible applications. In the paper [6], advantages of fractional-order controller for systems with time variable delays are presented and illustrated by examples. Up to the present time the problem of controllability for continuous-time and discrete-time linear dynamical systems has been extensively investigated in many papers and different controllability conditions are well known. However, this is not true for the semilinear or generally for nonlinear dynamical control systems, especially with different types of delays in control and state variables.

Moreover, it should be pointed out, that since for nonlinear or semilinear dynamical systems the exact analytical form of the solutions of state equations is not known, it is necessary to use special mathematical methods in controllability problems [4,7–9]. Therefore, in the proofs of controllability results for nonlinear and semilinear dynamical systems, different methods such as linearization methods or fixed-point techniques are extensively used [7,10,11]. Fixed-point technique offers many fixed-point theorems such as for example: the Banach fixed-point theorem, Schauder fixed-point theorem, or the Rothe fixed-point theorem [12]. It should be stressed that using fixed-point techniques only sufficient, but not necessary controllability conditions may be formulated and proved. Let us recall that semilinear dynamical control systems contain linear and pure nonlinear parts in the differential state equations.

Mathematical models of real dynamical systems often contain delays in state variables or in controls. In the theory of dynamical systems with delays we may consider many different kinds of delays both in state variables or in admissible controls, e.g., distributed delays, point time-variable delays, point constant delays. Thus, in dynamical systems with delays it is necessary to introduce two kinds of state, namely: a complete state and an instantaneous state. Hence, we have two types of controllability: absolute controllability for complete states and relative controllability for instantaneous states [1,13].

In the last few years, the fixed-point technique has often been used to formulate and prove sufficient conditions for controllability of semilinear and generally nonlinear dynamical control systems. The main purpose of this chapter is to study the relative controllability of semilinear control systems with multiple variable delays in admissible controls using Rothe's [12,14] fixed-point theorem.

The main purpose of this paper is to study the relative controllability of semilinear control systems with multiple variable delays in admissible controls using Rothe's fixed-point theorem. Finally, it should be pointed out, that the sufficient controllability conditions formulated and proved in the paper are a generalization to the many variable delays in the control case of the results recently published in [14].

2. System Description

Let us consider delay dynamical control systems containing multiple lumped time varying delays in admissible controls described by the following ordinary vector differential state equation

$$x'(t) = Ax(t) + \sum_{i=0}^{i=M} B_i u(v_i(t)) + f(x(t), u(v_0(t)), u(v_1(t)), \ldots, u(v_i(t)), \ldots, u(v_M(t))) \quad \text{for} t \in [v_M(t_0),\ t_1] \tag{1}$$

where:

$x(t) \in R^n$ is a pseudo or instantaneous state vector,
$u \in L^2_{loc}([t_0, \infty), R^p)$ is an admissible control,
A is $(n \times n)$-dimensional constant matrix with real elements,
B_i are $(n \times m)$-dimensional constant matrices with real elements for $i = 0, 1, 2, \ldots, M$,
f is the continuous mapping $f : R^n \times R^p \times R^p \times \ldots \times R^p \times \ldots \times R^p \to R^n$

such that $f(0, 0, 0, \ldots, 0, \ldots 0) = 0$, and there are real constants a, b, c, d such that $1/2 \le d \le 1$ and function f satisfies the inequality

$$\|f(x(t), u(v_0(t)), u(v_1(t)), \ldots, u(v_i(t)), \ldots, u(v_M(t)))\|_{R^n} \le a\|x(t)\|_{R^n} + b\|u(t)\|^d_{R^m} + c$$

where

$$\|u(t)\|_{R^m} = \sum_{i=0}^{i=M} \|u(v_i(t))_{\|R^m} \tag{2}$$

and $\|u(v_i(t))\|_{R^p}$, $i = 0, 1, 2, \ldots, M$ is a norm in the Hilbert space $L^2([v_M(t_0), t_1), R^p)$.

The strictly increasing and twice continuously differentiable functions $v_i(t)$: $[t_0,t_1] \to R$, $i = 0, 1, 2, \ldots, M$, represent deviating arguments in the admissible controls and in the state variables, i.e., $v_i(t) = t - h_i(t)$, where $h_i(t)$ are lumped time varying delays for $i = 0, 1, 2, \ldots, M$. Moreover, $v_i(t) \le t$ for $t \in [t_0,t_1]$, and $i = 0, 1, 2, 3, \ldots, M$.

Let us introduce the so called time-lead functions $r_i(t)$:$[v_i(t_0),v_i(t_1)] \to [t_0,t_1]$, $i = 0, 1, 2, 3, \ldots, M$, such that $r_i(v_i(t))$ = t for $t \in [t_0,t_1]$. Furthermore, only for simplicity and compactness of notations, let us assume that $v_0(t) = t$ and for a given time t_1 the functions $v_i(t)$ satisfy the following inequalities.

$$h = v_M(t_1) \le v_{M-1}(t_1) \le \ldots \le v_{m+1}(t_1) \le t_0 = v_m(t_1) < v_{\mathrm{m}-1}(t_1) \le \ldots \le v_1(t_1) \le v_0(t_1) = t_1 \tag{3}$$

For delayed dynamical control systems it is necessary to introduce at least two general kinds of states, namely: an instantaneous (or pseudo) finite dimensional state $x(t) \in R^n$, and a complete (or function) infinite dimensional state $z(t) = \{x(t), u_t(s)\}$, where $u_t(s) = u(s)\ for\ s \in [v_M(t), t)$. Moreover, it should be pointed out, that only the complete state $z(t)$ completely describes the behavior of the control system at a given time t.

Let $h \ge t_0 - v_M(t_0) > 0$ be given. For a given function x:$[t_0 - h,t_1] \to R^n$ and $t \in [t_0,t_1]$, the symbol x_t usually denotes the function on $[-h,0]$ defined by $x_t(s) = x(t + s)$ for $s \in [-h,0]$.

Similarly, for a given control function u:$[v_M(t_0),t_1]\rightarrow R^p$, and $t \in [t_0,t_1]$, the symbol u_t denotes the function on $[v_M(t),t)$ defined by the equality $u_t(s) = u(t + s)$ for $s \in [v_M(t),t)$. For example, u_{t_0} is the initial control function defined on time interval $[v_M(t_0),t_0)$.

Since for delayed systems there are two different concepts of states, then as a consequence there are two main different definitions of controllability: namely relative (weak) controllability and absolute (strong) controllability (see e.g., [1,2,15] for more details). In the case of relative controllability on $[t_0,t_1]$, the aim is to find an admissible control so that the instantaneous state $x(t_1)$ can be reached using this admissible control.

In the case of absolute controllability, the aim is to reach complete state. For dynamical systems with delays only in control it means that the final segment of an admissible control over the time interval $[v_M\ (t_1),t_1)$ should be a given m-dimensional function.

3. Preliminaries

Since in the paper the semilinear dynamical system is considered, the fixed-point technique will then be used to formulate and prove the main result of the paper, which is a sufficient condition for relative controllability in a given time interval. Among different fixed-point theorems presented in the literature, Rothe's fixed-point theorem is applied. For completeness of considerations, let us now recall Rothe's fixed-point theorem.

Rothe's fixed-point theorem [12,14]. Let E be a Banach space and B be a closed convex subset of E such that zero of E is contained in the interior of B. Let $g : B \rightarrow E$ be a continuous mapping with $g(B)$, relatively compact in E and $g(\partial B)$ is a subset of ∂B, where ∂B denotes the boundary of B. Then, there is a point $x^* \in B$ such that $g(x^*) = x^*$.

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In the case of absolute controllability, the aim is to reach a complete state. For dynamical systems with delays only in control it means that the final segment of an admissible control over the time interval $[v_M\ (t_1),t_1]$ should be a given m-dimensional function.

Since in the next parts of the paper only relative controllability is considered, now we recall definitions of attainable or reachable instantaneous states and relative controllability.

Definition 1 [1,15]. *The attainable set at time $t_1 > t_0$ from the given initial complete state $z(t_0) = \{x(t_0), u_{t_0})$ for the time delay control system (1) is the set as*

$$K([t_0, t_1]) = \{x \in R^n : x = x(t_1)\}$$

where x(t,u), is the solution of the state Equation (1) given by

$$\begin{aligned} x(t,u) = exp(At)x_0 + \int_{t_0}^{t} exp\, A(t-s) \sum_{i=0}^{i=M} B_i(s)u(v_i(s))ds+ \\ +\int_{t_0}^{t} exp\, A(t-s) f(x((s),\ u(v\ _0(s)), u(v_1(s)), \ldots, u(v_i(s)), \ldots, u(v_M(s)))ds \quad for\ \ t \in [t_0, t_1] \end{aligned} \tag{4}$$

with an initial complete state $z(t_0) = \{x(t_0),u_{t_0})$.

Since in the next parts of the paper only relative controllability is considered, now we recall definitions of relative controllability.

Definition 2. *The semilinear control system* (1) *is called relatively controllable on a given time interval* $[t_{0,}t_1]$ *if for each initial complete state* $z(t_0) = \{x(t_0), u_{t_0})$ *and in each final instantaneous state* $x_1 \in R^n$, *there exists an admissible control* $u_1 \in L^2([t_0, t_1], R^p)$ *such that* $x(t_1,u_1) = x_1$.

In other words, a semilinear system (1) is relatively controllable on given time interval $[t_{0,}t_1]$ if the attainable set $K([t_{0,}t_1])$ is the whole space R^n.

In the next part of the paper, in the proof of sufficient condition for relative controllability, the following estimation of the solution $x(t,u)$ is used:

$$\|x(t,u)\| \leq \left(\int_{t_0}^{t_1} \|B\| Qexp(q(t_1 - s))\|u(s)\|ds + \int_{t_0}^{t_1} b\ Qexp(q(t_1 - s))\|u(s)\|^c ds \right) exp(aQt_1)$$

where

$$\|B\| = \sum_{i=0}^{I=M} \|B_i\| = \sum_{i=0}^{i=M} max\left|b_{ikj}\right|,\ 1\ \leq\ k\ \leq\ n,\ 1\ \leq\ j\ \leq\ p$$

Next, using transformation and definite integral properties, the solution (4) of the Equation (1) can be presented in the following more convenient form:

$$\begin{aligned} x(t_1, z(0), u) &= exp(A(t_1 - t_0))q(z(t_0)) + \int_{t_0}^{t_1} exp(A(t_1 - s))B_{t_1}(s)u(s)ds + \\ &+ \int_{t_0}^{t_1} exp(A(t_1 - s))f(x(t), u(v_0(t)), u(v_1(t)), \ldots, u(v_i(t)), \ldots, u(v_M(t)))ds \end{aligned}$$

where

$$\begin{aligned} q(z(t_0)) &= x(t_0, u_0(s)) \\ &+ ((expA(t_1 - t_0)))^{-1} \left(\sum_{i=0}^{i=m} \int_{v_i(t_0)}^{t_0} exp(A(t_1 - v_i(s))B_i u_0(s)ds \right. \\ &\left. + \sum_{i=m+1}^{i=m} \int_{t_0}^{v_i(t_0)} exp(A(t_1 - v_i(s))B_i u_0(s)ds \right) B_{t_1}(s) \\ &= exp(A(t_1 - s)) \sum_{j=0}^{j=i} exp(A(t_1 - v_i(s)))B_j \end{aligned}$$

for $t \in [v_{i+1}(t_1), v_{i+1}(t_1))$, $i = 0, 1, 2, \ldots, m - 1$.

In the sequel we shall also consider the linear control system with multiple lumped time varying delays in the admissible controls and without delays in the state variables, described by the following differential state equation,

$$x'(t) = A + \sum_{i=0}^{i=M} B_i u(v_i(t)) \tag{5}$$

where A is $n \times n$ dimensional matrix with real elements.

Using the time lead functions and the inequalities (3) we have

$$\begin{aligned} x(t_1, u) &= exp\ A(t_1 - t_0)x_0 + \sum_{i=0}^{i=m} \int_{v_i(t_0)}^{t_0} exp\ A(t_1 - r_i(s))B_i r'_i(s)u_{t_0}(s)ds + \\ &+ \sum_{i=m+1}^{i=M} \int_{v_i(t_0)}^{v_i(t_1)} exp\ A(t_1 - r_i(s))B_i r'_i(s)u_{t_0}(s)ds + \\ &+ \sum_{i=0}^{i=m} \int_{t_0}^{t_1} exp\ A(t_1 - r_i(s))B_i r'_i(s)u(s)ds \end{aligned} \tag{6}$$

For brevity of considerations, let us introduce the following commonly used notations

$$
\begin{aligned}
x(t_1) = exp\, A(t_1 - t_0)x_0 + \sum_{i=0}^{i=m} \int_{v_i(t_0)}^{t_0} exp\, A(t_1 - r_i(s))B_i r'_i(s) u_{t_0}(s)ds + \\
+ \sum_{i=m+1}^{i=M} \int_{v_i(t_0)}^{v_i(t_1)} exp\, A(t_1 - r_i(s))B_i r'_i(s) u_{t_0}(s)ds + \\
+ \sum_{i=0}^{i=m} \int_{t_0}^{t_1} exp\, A(t_1 - r_i(s))B_i r'_i(s) u(s)ds
\end{aligned} \tag{7}
$$

Thus, $H(t, u_{t_0}) \in R^n$

$$
\begin{aligned}
x(t_1) = exp\, A(t_1 - t_0)x_0 + \sum_{i=0}^{i=m} \int_{v_i(t_0)}^{t_0} exp\, A(t_1 - r_i(s))B_i r'_i(s) u_{t_0}(s)ds + \\
+ \sum_{i=m+1}^{i=M} \int_{v_i(t_0)}^{v_i(t_1)} exp\, A(t_1 - r_i(s))B_i r'_i(s) u_{t_0}(s)ds + \\
+ \sum_{i=0}^{i=m} \int_{t_0}^{t_1} exp\, A(t_1 - r_i(s))B_i r'_i(s) u(s)ds
\end{aligned} \tag{8}
$$

Hence, using (7) we have

$$
\begin{aligned}
q(t_1, u_{t_0}) = exp\, A(t_1 - t_0)x_0 + \sum_{i=0}^{i=m} \int_{v_i(t_0)}^{t_0} exp\, A(t_1 - r_i(s))B_i r'_i(s) u_{t_0}(s)ds + \\
+ \sum_{i=m+1}^{i=M} \int_{v_i(t_0)}^{v_i(t_1)} exp\, A(t_1 - r_i(s))B_i r'_i(s) u_{t_0}(s)ds +
\end{aligned} \tag{9}
$$

Thus, $q(t_1, u_{t_0}) \in R^n$.
Let us denote

$$
G_m(t,s) = \sum_{i=0}^{i=m} exp\, A(t_1 - r_i(s))B_i r'_i(s) \tag{10}
$$

Thus, $G_m(t,s)$ is an $n \times p$ dimensional matrix.

4. Controllability Conditions

First of all, let us consider the linear control system described by the differential the state Equation (5). Using the standard methods (see e.g., [1,15] for more details), let us define the $n \times n$ dimensional relative controllability matrix $W_0(t_0,t_1)$ for the linear dynamical control system (5):

$$
W(t_0, t_1) = \int_{t_0}^{t_1} G_m(t_1, s) G_m^{tr}(t_1, s) ds \tag{11}
$$

where tr denotes matrix transpose.

Therefore, the relative controllability matrix $W(t_0,t_1)$ is the $n \times n$ dimensional symmetric matrix depending only on time interval $[t_0,t_1]$ and system parameters.

Now, let us recall the most frequently used necessary and sufficient condition for relative controllability.

Theorem 1 [6,7]. *The linear delayed system (5) is relatively controllable in the given time interval* $[t_0,t_1]$ *if and only if*

$$rank\ W(t_0,t_1) = n$$

Now, let us consider relative controllability conditions for semilinear control systems with differential state Equation (1). For simplicity of considerations let us introduce the following notations:

$$G(u) = \int_{t_0}^{t_1} expA(t_1 - s)B_{t_1}(s)u(s)ds$$

$$GG^* = \int_{t_0}^{t_1} expA(t_1 - s)B_{t_1}(s)B^*_{t_1}(s)expA^*(t_1 - s)ds$$

$$H(u) = \int_{t_0}^{t_1} expA(t_1 - s)f(x_u(t), u(v_0(t)), u(v_1(t)), \ldots, u(v_i(t)), \ldots, u(v_M(t)))ds$$

where $x_u(t)$ is the solution of Equation (1) for the control $u(t)$.

Thus, H: $L^2([t_0, t_1], R^p) \to R^n$ is the nonlinear operator.

Moreover, the controllability operator

$$G_f(u) = G(u) + H(u)$$

Let us consider adjoint operator $G^*\ : R^n \to L^2([t_0, t_1], R^p)$ of the operator G given by the formula:

$$G^*(x) = B^*_{t_1}(s)exp(A^*(t_1 - s))x,\ for\ s \in [t_0 - t_1]$$

Now, taking into account Theorem 1, relative controllability matrix $W_0(t_0,t_1)$ and, using Rothe's fixed-point theorem, the following sufficient condition for relative controllability on $[t_0,t_1]$ may be formulated and proved.

Theorem 2. *If the linear control system (5) is relatively controllable on* $[t_0,t_1]$ *and the following inequality holds*

$$\left(g\ \sqrt{2}\right)^{-1}\|B\|^2Q^3\ a\ \sqrt{t_1}exp(aQt_1)q^{-1.5}(exp2qt_1 - 1)^{1.5} < 1$$

then the semilinear control system (1) is also relatively controllable on $[t_0,t_1]$.

Moreover, an admissible control steering the dynamical control system (1) from the given initial complete state $z(t_0)$ to a given final instantaneous state $x_1 = x(t_1)$ at time $t_1 > t_0$ is given by the formula

$$u(t) = B^*_{t_1}(s)exp\Big(A^*(t_1 - s)\Big)(GG^*)^{-1}(x_1 - exp(A(t_1 - t_0)\Big)q(z(t_0) - H(u)),\ t\epsilon[t_0, t_1]$$

Proof. Following [9,16] the proof is based on the fixed-point technique, i.e., on Rothe's fixed-point theorem. The crucial point is to define the nonlinear operator P. Thus, for each fixed vector $x(t) \in R^n$ we define the nonlinear operator

$$P:\ L^2([t_0, t_1]), R^p) \to L^2([t_0, t_1]), R^p)$$

given by the formula

$$P(u) = \ G^*(GG^*)^{-1}(x - H(u))$$

Since the operator $(GG^*)^{-1}$ exists, then the nonlinear operator P is well defined and, moreover, the following inequality holds

$$||(GG^*)^{-1}x|| \leq g^{-1}||x||$$

□

Now using Rothe's fixed-point theorem [12,14] we shall prove that the operator P has a fixed-point u that depends on instantaneous state x.

First of all, let us observe that since function f is continuous, then operator P is also continuous and, due to inequality (2), it is a compact operator.

For $u \in L^2([t_0, t_1]), R^p)$ applying Hőlder inequality and condition (2), the following estimation follows:

$$||H(u)|| \leq \int_{t_0}^{t_1} Qexp(q(t_1 - s))f(x_u(s),)d\,u(v_0(s)), u(v_1(s), \ldots, u(v_i(s)), \ldots, u(v_M(s))ds \leq$$

$$\leq \left(\int_{t_0}^{t_1} Q^2 exp(2q(t_1 - s)ds\right)^{\frac{1}{2}} \times$$

$$\times \int_{t_0}^{t_1}\Big(||f(x_u(s), dsu(v_0(s)), u(v_1(s), \ldots, u(v_i(s)), \ldots, u(v_M(s))||^2 ds\Big)^{\frac{1}{2}} \leq$$

$$\leq N\left(\int_{t_0}^{t_1}\Big(a||x(s)|| + b||u(s)||^c\Big)^2 ds\right)^{\frac{1}{2}}$$

$$\leq N\left(\int_{t_0}^{t_1}\Big(4a^2||x(s)||^2 + 4b^2||x(s)||^{2c}\Big)^2 ds\right)^{\frac{1}{2}} \leq$$

$$\leq 2Na\left(\int_{t_0}^{t_1}||x(s)||^2 ds\right)^{\frac{1}{2}} 2Nb\left(\int_{t_0}^{t_1}\Big(||x(s)||^{2d}\Big)^2 ds\right)^{\frac{1}{2}} \leq$$

$$\leq 2N^2 aexp((t_1 - t_0)aQ)\Big(\sqrt{t_1 - t_0}||B||||u||_{L^2}$$

$$+ +2Nb\Big(t_1 - t_0\Big)^{0.5(1-d)}\Big(Naexp(aQ(t_1 - t_0))\sqrt{t_1 - t_0}$$

$$+ 1)\Big(||u||_{L^{2d}}\Big)^d$$

where constant N can be computed as follows:

$$N = \left(\int_{t_0}^{t_1} Q^2 exp(2q(t_1 - s)ds\right)^{\frac{1}{2}}$$

Thus, for $||u||_{L^2}$ and $\frac{1}{2} \leq d \leq 1$ the following inequality follows

$$||P(u)||_{L^2} \leq r||u||_{L^2}$$

where by the assumption

$$r = \Big(g\sqrt{2}\Big)^{-1}||B||^2 Q^3 a\sqrt{t_1} exp\Big(aQt_1\Big(q^{-1.5}(exp(2qt_1 - 1))\Big)^{1.5} < 1$$

Therefore, for a fixed ε, $r < \varepsilon < 1$, there exists $r_0 > 0$ such that

$$||P(u)||_{L^2} \leq \varepsilon||u||_{L^2} = \varepsilon r_0$$

Let $B(0,d_0)$ denote the ball centered at zero and with radius $d_0 > 0$ and boundary $\partial B(0,d_0)$. Then, from the above inequality, it follows that $P(\partial B(0,d_0)) \in \partial B(0,d_0)$. Thus, nonlinear operator P is a compact operator and maps the sphere $\partial B(0,d_0)$ into the interior of the ball $\partial B(0,d_0)$.

Therefore, Rothe's fixed-point theorem can be applied and there exists an admissible control u depending on x such that

$$u = P(u) = \ G^*(GG^*)^{-1}(x - H(u))$$

Finally, from the above, it follows that the admissible control steering the system (1) from the initial complete state $z(t_0) = \{x(t_0),u_{t_0})$ to a final instantaneous state $x_1 = x(t_1)$ at time $t_1 > t_0$ is given by the following formula

$$u(t) = B^*_{t_1}(s)exp\Big(A^*(t_1 - s)(GG^*)^{-1}(x_1 - exp(A(t_1 - t_0)\Big)q(z(t_0) - H(u))\ t\epsilon[t_0, t_1]$$

5. Conclusions

In the paper, using Rothe fixed-point theorem, sufficient conditions for relative unconstrained controllability of semilinear control systems with multiple time varying point delays in an admissible control were formulated and proved. Moreover, it should be pointed out that the obtained results can be extended at least in many directions, namely, for integrodifferential control systems [13,17,18], for fractional semilinear control systems [14,16,19], and for semilinear standard systems with distributed delays in an admissible control [15].

Finally, it should be pointed out, that some of the ideas of the work can be used to study the controllability of a control system governed by semilinear fractional differential equations presented recently in monograph [15] or by diffusion processes defined on a bounded domain and discussed in the paper [1]. Controllability of semilinear fractional control systems needs additional results taken directly from nonlinear functional analysis.

Funding: This research received no external funding.

Acknowledgments: The research was founded by Polish National Research Centre under grant "The use of fractional order controllers in congestion control mechanism of Internet", grant number UMO-2017/27/B/ST6/00145.

Conflicts of Interest: The author declares no conflict of interest.

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