Markov Models for Dimensioning and Provisioning of Battery Energy Storage Systems (BESS) for Off-Grid Green Mobile Network Base Station Sites

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Abstract

There is a growing desire by mobile network operators and other stakeholders to reduce carbon emissions from the operation of mobile networks. The widespread adoption of ultra-dense 5G and Internet of Things (IoT) networks will likely increase the energy demand from these networks significantly. In this paper, we model the density of the time required to charge BESS to its full capacity when the renewable energy sources can generate enough energy to meet the needs of the base station site and the density of the time required to completely deplete the energy stored in BESS when the renewable energy sources cannot generate a sufficient amount of energy to meet the demand of the site. We also investigate the influence of the design parameters, such as the energy supply-demand ratio, on the distribution of the time required to charge BESS to its full capacity (for a supply-demand ratio greater than one) and the time required to completely deplete the energy stored in BESS (for supply-demand ratio less than one).

Keywords: Markov Models; dimensioning of green networks; battery energy storage systems (BESS); green mobile networks.

1 Introduction

Mobile communication technologies are playing significant role in the socio-economic development of every society worldwideGelenbe and Abdelrahman (2018). Their widespread adoption has led to the deployment of massive numbers of base station sites to handle the growing number of users, increasing the carbon footprint, especially in off-grid areas where base station sites are powered using diesel generators. Stochastic models such as Markovian models have been applied to size Battery Energy Storage Systems (BESS) for Green base station sites in mobile networks. These models are based on the discretisation or quantisation of energy delivered to BESS into energy packets and then analysed using well-known Markovian modelling methods used to analyse

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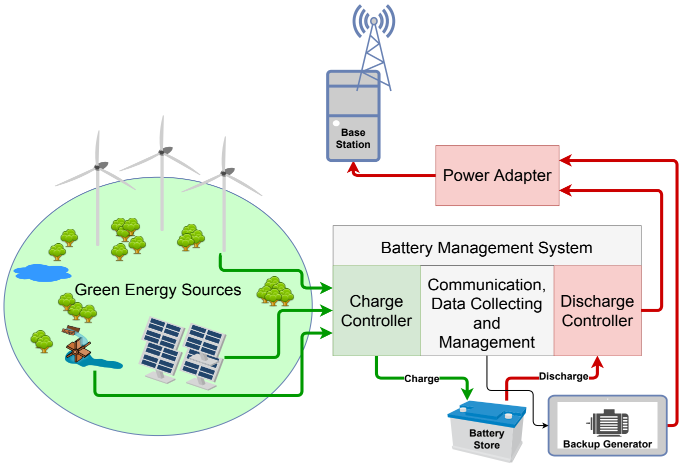

Figure 1: The architecture of a green base station site.

computer networks. An alternative approach is to use a diffusion process to represent the energy content of BESS and its dynamic evolution over time as inCzachórski et al. (2022b); Kuaban et al. (ress); Czachórski et al. (2022a).

The energy packet-based Markov model for battery energy storage systems for green base station sites supplied by renewable energy sources was discussed in Gelenbe and Abdelrahman (2018). In this paper, we model the density of the time required to charge BESS to its full capacity when the renewable energy sources can generate enough energy to meet the needs of the base station site and the density of the time required to completely deplete the energy stored in BESS when the renewable energy sources cannot generate a sufficient amount of energy to meet the demand of the site.

2 Markov models for battery energy storage system

Consider a base station site that is powered using energy generated from renewable energy sources such as solar or wind energy systems. If this energy is greater than the energy demand of the base station site, the Battery energy storage (BESS) system is charged. When the renewable energy system cannot generate sufficient energy to supply the site, the battery energy storage system is discharged to supply the site. If the battery energy storage system is completely discharged, the backup generator picks up (although the cost of running the generator is high, it prevents the site from shutting down).

Suppose that initially, we have i energy packets (for $i \in [1,\ B]$) in BESS at time $t = 0$. Also, let the random variable T represent the time after which the battery becomes empty, which is the first passage time of the process from $N(0) = i$ to $N(t) = 0$ (when all the energy stored in the battery is completely depleted), that is $T =\in f\{t > 0 : N(t) = 0 \mid N(0) = i\}$. The density of the time required to deplete the energy stored in BESS completely is modelled by the density of the time required for the Markov process that starts at a point i to reach 0 (see, e.g. Takagi and Tarabia (2009)). Hence, the Laplace transform of density of the distribution of the time required to deplete the energy stored in BESS completely is:

$$\bar{h}_{i,0}(s) = \varrho^{-i}\frac{[\eta(s)]^{B-i}[\eta(s)-1]+[\xi(s)]^{B-i}[\xi(s)-1]}{[\eta(s)]^{B}[\eta(s)-1]+[\xi(s)]^{B}[\xi(s)-1]} \tag{1}$$

where,

$$\xi(s) = \frac{s+\lambda+\mu-\sqrt{(s+\lambda+\mu)^2-4\lambda\mu}}{2\lambda},$$
$$\eta(s) = \frac{s+\lambda+\mu+\sqrt{(s+\lambda+\mu)^2-4\lambda\mu}}{2\lambda},$$

and $\varrho = \lambda/\mu$ is the energy supply-demand ratio. Also, assuming that initially, the BESS is fully charged ($i = B$), then the first passage time from $N(0) = B$ to $N(t) = 0$ is

$$\bar{h}_{B,0}(s) = \varrho^{-B}\frac{[\eta(s)-1]+[\xi(s)-1]}{[\eta(s)]^{B}[\eta(s)-1]+[\xi(s)]^{B}[\xi(s)-1]} \tag{2}$$

and for oversized or large battery capacity,

$$\lim_{B\to\infty}\bar{h}_{i,0}(s) = [\xi(s)]^{i} \tag{3}$$

It is essential to know the time required to charge BESS to its full capacity when the renewable energy sources can generate enough energy to meet the energy demand of the site and store extra energy in BESS to be used later when the renewable energy sources are not able to generate sufficient energy to meet the demand of the site. The time required to charge BESS for an initial amount of energy i to its full capacity is given in the Laplace domain and can be adapted from equation (1) as (we cite the result with a minor correction to the original Takagi and Tarabia (2009) p.223: ϱ is replaced by ϱ^{B-i}):

$$\bar{h}_{i,B}(s) = \varrho^{-(B-i)}\frac{\{[\eta(s)]^{i+1}-[\xi(s)]^{i+1}\}-\{[\eta(s)]^{i}-[\xi(s)]^{i}\}}{\{[\eta(s)]^{B+1}-[\xi(s)]^{B+1}\}-\{[\eta(s)]^{B}-[\xi(s)]^{B}\}} \tag{4}$$

Assuming that BESS was initially empty (i = 0), then the first passage time of the process from $N(0) = 0$ to $N(t) = B$ is

$$\bar{h}_{0,B}(s) = \varrho^{-B}\frac{[\eta(s)-\xi(s)]-[\eta(s)-\xi(s)]}{\{[\eta(s)]^{B+1}-[\xi(s)]^{B+1}\}-\{[\eta(s)]^{B}-[\xi(s)]^{B}\}} \tag{5}$$

For very large battery capacity,

$$\lim_{B\to\infty}\bar{h}_{i,B}(s) = [\xi(s)]^{B-i} \tag{6}$$

3 Results

In the numerical results presented, we consider that the capacity of BESS is $B = 100$ KWh. Typically, an average energy demand of a based station site required to power all the equipment in the site is between 1 KW and 3 KW Mohamad Aris and Shabani (2015). In this paper, we consider the site's energy demand of $\mu = 1$ KW. The energy supply-demand ratio, ϱ, is varied by varying the mean energy supply rate λ leaving the energy demand rate, μ constant.

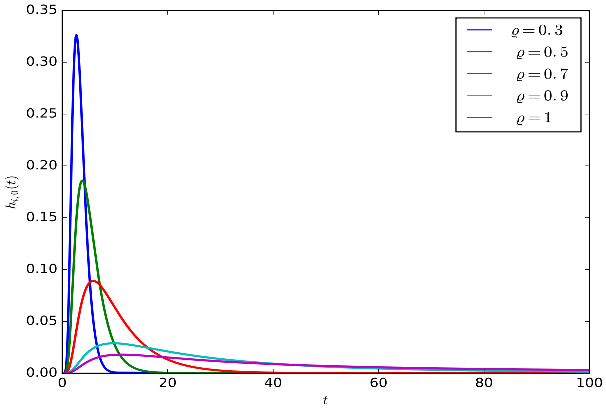

Figure 2: The influence of ϱ on the probability density of the time after which the energy stored in the BESS is completely depleted, $h_{i,0}(t)$, for $\lambda = \{0.3, 0.5, 0.7, 0.9, 1\}$, $\mu = 1$, $i = 10$, $\bar{h}_{i,0}(s)$ given by eq(1) numerically inverted.

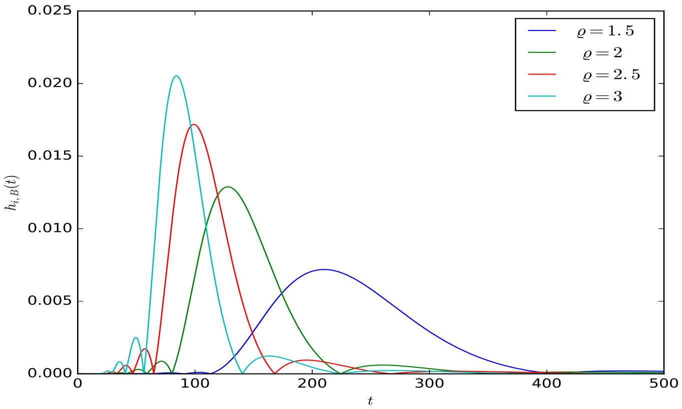

Figure 3: The influence of ϱ on the probability density of the time after which the BESS is fully charged, $h_{i,B}(t)$, for $i = 10$, $\lambda = \{1.5, 2, 2.5, 3\}$, $\mu = 1$, $B = 100$, $\bar{h}_{i,B}(s)$ given by eq(4) numerically inverted.

Figure 3 shows the influence of ϱ on the probability density of the time, after which the energy stored in the BESS is completely depleted. It can be seen that the probability of the time after which the energy stored in the BESS is completely depleted decreases with increasing ratio of the energy supplied to the BESS to the energy drawn, ϱ. Similarly, the influence of the energy supply-demand ratio, ϱ on the density of the distribution of the time required to charg BESS to its full capacity is shown in Figure 3.

4 Conclusion

The results presented in this study shows that by increasing the energy supply-demand ratio ϱ, the probability of completely depleting the energy stored in BESS is decreased, increasing the reliability and the energy self-sustainability of the site. Also, It can be observed that as ϱ increases, the time required to charge BESS to its full capacity increases. We intend to extend this study by investigating the influence of other design parameters.

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