Doctoral Dissertation Review

Bratislava, 4/10/2017

Przemysław Sadowski's thesis *Quantum Walks: various models and their algorithmic applications* looks at various approaches to quantum walks (QW). The systems in question are quantum particles that can move around or send signals, especially in coherent superpositions – for example an excitation that hops or spreads in a spin chain. In general, a QW takes a quantum system and evolves it by unitary transformations (standard coined quantum walks) or quantum channels (open quantum walks). After some time, we can stop the process and make a measurement. This way, we can obtain random results that we can compare to those obtainable by classical random walks (e.g. Markov Chains). These classical walks are a mathematical tool utilizing random processes for algorithmic applications – searching, network exploration, signal transmission, volume calculation, sampling, etc.

After an intro to quantum walks, the thesis presents a collection of independent results, spread over 5 chapters (CH4-CH8). In each, we find a mix of generalization of particularity. The author picks a model of a quantum walk, generalizes it, and then usually chooses a particular scenario (e.g. a graph, or a game), where he proves interesting novel results about how quantum walk scenario differ from their classical analogues. The results include a strange limiting distribution, a way to beat a network-exploring game, a speedup in search, the impact of the environment on network dynamics, and central limit theorems.

These results have been published in 4 papers and 1 preprint, listed on page i. Overall, during his doctoral studies, the author has produced 11 peer-reviewed papers and 3 preprints. This is more than enough production required for a doctoral degree candidate.

The quality of the thesis writing itself is far from excellent, and in my opinion would benefit from rereading, editing and some additional writing. The readability of the thesis could thus be greatly improved. First, by fixing the grammar (e.g. innumerable missing or superfluous *a*'s and *the*'s), typographical errors (small brackets, straight quotation marks), and sentences with repetitive words by more careful proofreading. Second, and much more important, the intros to the chapters need a bit more connecting material, as now the paragraphs and sentences are often quite disconnected, jumping from general remarks to the author's results and back. These results need to be introduced in a gentler way and emphasized. The reader should not need to scan the bibliography to see which of the references are the author's papers. The reader really has to wait until the conclusions in Chapter 9 for a summary of the results. I just now wish I have read it first.

Nevertheless, the content of the thesis is a nice collection of independent and novel research work that deserves the interest of quantum information scientists. The problems with the writing mentioned above can easily be remedied by some editing and a solid presentation and a Q&A session at the thesis defense.

Therefore, *I recommend the committee to grant a doctoral degree* to Przemysław Sadowski for his solid and innovative work on the many facets of quantum walks.

Mgr. Daniel Nagaj, PhD.

independent researcher
Research Center for Quantum Information
Institute of Physics
Slovak Academy of Sciences
Dúbravská cesta 9
845 11 Bratislava
Slovakia

daniel.nagaj@savba.sk
+421 949 587 270

General, Chapter by Chapter Commentary and Questions

CH1 Intro

The first chapter is a quick intro and motivation for investigating quantum walks, and a sketch of what the author will do – investigate possible distributions, implementations of search/exploration tasks, etc., and compare them to classical possibilities. I only found the discussion of quantum games is a bit disconnected from the material around it. Finally, I wish the author was more precise in expressions like "*classical-simulable and strictly-quantum information processing systems*". Do you mean they are classically *not* efficiently simulable or something similar?

CH2 Preliminaries

The chapter on preliminaries is a mix of definitions, with rare glimpses of connections to the actual thesis, where it will be used, etc. I don't recommend an uninitiated reader to start with this Chapter. To suggest a direction for improvement, let me present a few examples. When discussing entanglement, you should say more about why it is important, how it is defined for mixed states, and discuss more about its applications or its properties as the killer of possible efficient simulation. In the general discussion of quantum walks, I wish for a connection to your thesis – why are walks not simulable? Is it because they are entangled? How do things scale, will it not be because of N walkers? Also, when talking about quantum systems and randomness, did you mean that a closed system has no classical randomness, while you could generate randomness by quantum evolutions? Next, when you discuss channels, give an example first, e.g. I choose 50% one unitary, 50% another. Then give the definitions, like you do e.g. in (2.42). You are not consistent in this. Finally, when you discuss measurements, you could also add a note connecting this with walks – evolve unitarily, measure = get randomness, a possible transition from quantum to classical = measure too often.

CH3 Intro to Quantum Walks

When introducing concepts, make a note on how it connects to walks. On page 22, you could say what kinds of expectation values you will be looking at (position, time of arrival, etc.). The introduction of martingales on p23 was too quick for me – why? What are they in brief? I got more from reading wikipedia on this. You can then also link to this intro when you start talking about them e.g. on p23.

3.2 Classical random walks

You say that random walks were historically studied without applications in mind. Is this true? Even so, so what? Are you trying to make a point that your walks do not need applications to be interesting?

On page 25 you talk about \sqrt{t} being standard behavior for classical random walks. This is true only on a line, no? What if the walk is modified into some direction? How is this general behavior? Isn't this just this SPECIFIC case?

3.3 Quantum walks

The author generalizes classical walks here. Why this way? Could you expand your small note about measuring often and the transition to a classical random process? I would also welcome a table comparing all the different quantum walk models, from least to most general, with their positives and negatives? It is also a bit unclear who the target audience is, and as an intro, it would benefit from illustrations, to see right away what the geometry is when you read through.

I have also noticed that in your generalization to coined walks, you don't talk about memory – but the state of a walk somehow remembers where it came from (it has the coin register). Is this the reason for weird properties? I wish for a discussion of this.

When Grover's walk comes in on page 30, you could note that the coin register has the same size as the position register, so you effectively introduce a new model (scattering walks) with two position registers that could come handy later. The state space is built from directed edges, and there's no need for a coin register. This is indeed what you do on page 32 – but sadly without any connection of what you already spelled out.

Perhaps it would be nice if you described the dynamics of Grover's walk without the oracle. Only then go into what happens when you introduce the oracle.

When you talk about walks on a lattice on page 31, it is not clear what your motivation is. You also talk about generalizations from 1D. Are you looking for other models that have ballistic behavior? Note that you say something like this but you haven't even mentioned ballistic behavior so far. Next, self-avoiding walks come simply out of the blue. Lazy walks surely exist also in 2D or on general lattices.

A picture would help depict what your coin does much better than (3.45). Then you talk about general graphs on page 33, and present the simplest shift operator consistent with the graph. You should also say that this is doing nothing but taking $|x, y\rangle$ to $|y,x\rangle$ – reflecting back and forth – a boring walk. Maybe you could use this point to rewrite a previously talked about walk on a lattice (1D or 2D) in this language.

Finally, on page 33 you finally talk about walks with memory. This would be a good point to look back at the Grover coin, which in some sense has memory – as for large n, it actually approaches your "simplest shift operator" from around (3.53), almost always reflecting a walk back, making it very lazy.

CH4 Long-range moves

This chapter on walks on a cycle with possible long range moves contains three results. First, the analysis of the spatial periodicity of time-averaged probability distributions of walks on such a modified cycle. Second, an analysis of adversarial coin-modifying trapping with a proof that it is impossible. Third, an analysis of behavior of this network with a missing link, and a way of detecting this.

It would be helpful to clearly state in the intro to the chapter which of the results are the author's work, so that the reader doesn't have to go to the bibliography to check this.

It would also help to have a more substantial intro on algorithms and quantum networks somewhere in the intro chapter, showing also where your results fit in and push the boundaries. Here it feels a bit rushed and disconnected from the work itself. Perhaps you could also motivate your choice of graph.

I have a few questions about the coin-trapping scenario. In 4.3, when you discuss fighting against trapping, what exactly is what the second player can do? Choose the coin once as some unitary? Why not measure or replace the coin register? Isn't it true, that the whole defense against trapping lies in the choice of a mixed-coin initial state? Couldn't a "reflecting" coin (your most trivial one from the previous Section) also "trap" the walk, now in $1 + 2.3 = 7$ nodes?

The images are often placed not on the page where they are talked about, especially in the Section on broken links. This could be easily fixed. It is also unclear to me why the number of required measurements should *decrease* with a growing liveliness parameter for a fixed, large N. You say the graph is then "more connected", and so a broken link has a smaller impact. Shouldn't then an OK graph and a graph with a broken link be *harder* to distinguish, and shouldn't the number of measurements *increase*?

The nice result of 4.4 says that even though the network is invariant under a shift by a single site, the author shows that the limiting, time-averaged distribution is periodic in space with a period equal to the liveliness parameter. Note that to avoid confusion, you should emphasize this periodicity is not in time.

Later, in Figure 4.4, it is unclear what "position" means. The location of the broken link or the measurement? What are you actually measuring? Moreover, is the initial state you've chosen optimal?

CH5 Modeling sense of direction

This chapter on the Magnus-Derek game is in my opinion the weakest part of the thesis, and I have many questions about it. It describes a game between two players, one, Magnus, choosing a magnitude, and another, Derek, choosing the direction of steps. The goal of Magnus is to reach all points in a network, while Derek wants to prevent this.

The chapter presents classical results on this game on a cycle, and then attempts to solve a quantum version of the game. I found the quantum version very weakly defined (visiting/attaining a vertex), with a single strategy for both goals, while it is unclear, what the rules of the game actually are. Thus, while interesting, I believe this part requires some work to make it stand on its own.

There are two quantum definitions of how one can reach a vertex– *attaining* or *visiting*. One involves probability 1 to be at a vertex at some time, while the other requires projective measurements of position (am I at vertex x or not) at each step. The strategy that was worked out here tries to answer both questions at the same time, which is unclear to me. Also, it is unclear whether (or why not) the position is measured after every step (and according to which projector). Next, it looks like your Magnus is using a quantized version of the classical optimal strategy, while it is unclear why this is the best thing for him to do quantumly. Is it because if he used superpositions, he would never "visit" or "attain" a node? What if he asked for a measurement once in a while?

Finally, why are you also considering the game for n-1 steps, wouldn't it be interesting to also consider more steps?

CH6 Search algorithms

This chapter investigates possibilities for quantum search algorithms that look at and utilize the network structure, and things that have been learned during the search. The algorithms presented here rely on a valid/invalid labeling of edges by a phase that is added when you take a quantum step on the edge.

First, on page 68, he shows how when a phase change is added to an edge, there is way to evaluate it by one step of a quantum walk combined with a Fourier transform. This is a basic tool for what follows.

Next, the author considers a scenario where only sqrt(N) edges (always a fraction on each level) on a k-regular tree are deemed "valid" (with some phases on them). Thus, we have info that the "valid" graph is a subgraph of the whole tree. He then presents a search algorithm that finds a marked vertex while walking only on these "valid" edges, which nicely beats a naive walk search that does not utilize (or try to extract) the valid/invalid edge information.

This scheme is then generalized on page 73 to more general graphs, and then on page 74 to multilevel graphs. In discussing these, I would prefer not to talk about speedups, but simply call this a clever way to go around a trap – when a step in a walk should or should not be taken – by evaluating whether an edge is valid or not, and only then taking a step.

On page 75, it is not clear what you mean with the case "where all the information encoded into the network is available".

CH7 Open quantum walks

This chapter introduces a new model of open quantum walks, where the transformations of a walker on an edge corresponds to some channels. It is supposed to model coinless walks with interaction with some reservoir. However, I am missing here a simple comparison/relationship to previous models. What is the motivation for these open walks? Is it a weaker generalization of classical random walks with only a bit of "quantum"? Are they more "realistic" or experimentally accessible? What kind of "noise" on top of regular quantum walks can they model? One place where one gets a little bit of insight comes way too late, at the end of a Section on the bottom of page 79 (if the internal dimension is 1, the model becomes equivalent to classical Markov chains). The intro to 7.2 and the top paragraph of page 82 provide some help with this, but again, it comes a bit late.

I wish I got an explanation or comparison of how the new models introduced (CH7-8) relate to the ones from the previous chapters (CH4-6).

In the intro, I miss an explanation/illustration of the scale-free and small-world concepts which you mention. Next, you also should illustrate the recursively-generated Apollonian networks right in the intro, don't leave it for Figure 7.4 that comes 11 pages later.

The intro should also contain a summary of results of the chapter, with clear designations of what is old, and what is new, what is proven and what is numerical observation (e.g. Remark 7.18). Next, plenty of definitions miss explanations on page 83. I wish you would help the reader here. I also wish for a better explanation of the role of the view operator that looks at/selects some property of the internal degree of freedom of the walker.

A fun example is studied on page 84. Then more examples for larger networks demonstrate nearly-classical and fully quantum featured properties for different initial settings and choice of operators. However, the choice of view operators was not cleanly motivated for me, I was not able to learn why the author chose this case to study.

The conclusion is an observation of non-trivial behavior. I wish the author would summarize and highlight the details of the results (e.g. using words like ballistic, periodic, visiting only a subset of vertices, arriving at a vertex at a given time etc., as is done later, and only very sideways). When talking about a comparison to classical behavior, you should summarize that too, don't expect the reader to know and deduce all. The thesis should be self-contained.

When you say that your GOQW can explain non-trivial behavior, this is not quite true, you have shown that it can have non-trivial behavior, but without a relationship to some model, with known behavior, which you want to model – e.g. by your GOQW. This is a weak point.

CH8 Central limit theorems for OQW

Here the author analyzes asymptotic behavior (large time limit) of Open quantum walks (OQW). There are two approaches present – modifying the walk to a homogeneous one (every vertex belonging to 1 class, these were previously analyzed), or analyzing the more complicated (inhomogeneous) case. First, the author finds rules where the homogeneous approach works. Second, he proves a new central limit theorem for OQW.

This is illustrated on nice examples. I would just like to ask, if this is the same model as in the previous chapter(s)? If not, what are the differences? Can there be coherent behavior here? What is quantum about this? The density matrices evolve according to a channel, depending on what vertex the walker is at. However, are superpositions of being at different vertices valid now?

The first result on reducible walks comes on page 100. It says that when a walker can end only in vertices of the same class after "l" steps, then we can combine these steps, and get a new, homogeneous walk from this. The author then shows a central limit theorem. An example for this is a bipartite graph – (a subset of) a 2D grid in Figure 8.1. I miss a motivation for the choice of (8.15, 8.16).

On page 103, we find the second result on nonhomogeneous graphs with several classes of vertices, assuming the probability of a vertex being in some class is the same regardless of the position.

CH9 Conclusions

At last, this Chapter gives us a higher-level view of all the results. This was a very welcome chapter to read. I only missed a bit of a note on the implementation of such walks in real life, if these are supposed to be as influential and important as the author claims (beyond just being a mathematical model that we can perhaps simulate classically).

There is one important sentence on page 113 about open quantum walks that have applications in quantum biology and dissipative quantum computing. I'd like to know more!

I have two final comments. In the summary of Chapter 5 you say that both players can significantly change the parameters of the exploration. What Derek can do here is confusing – what exactly he can prevent or not (especially because a reader reading just the conclusions can't have a clue what "visiting" and "attaining" means in your quantum scenario). The tradeoffs described here are also nebulous if one does not read Chapter 5 in detail.

Finally, in the summary of Chapter 7 about Apollonian networks, you say the results differ significantly from classical ones. Can you describe this qualitatively? Is it about speedups or impossibilities?

Specific Page by Page Comments and Questions

page 1

modeled gave predictions ... gave predictions

2

precessing ... processing

7

(2.16) the ket/bra on k is reversed

15 after (2.38)

You don't say what it means for the operator S to be "normal". Also, say why complete positivity is what you want for channels.

16

The act of measurement introduces randomness – note that this is true in most cases, but not in special cases, you should note this, e.g. when evolving and measuring eigenstates, etc.

20

You use A as a name of a set, while right in the previous Sections it used to be a unitary operator. Maybe consider using M_i in Section 2.4 instead?

22 (3.10)

Write a macro for the expectation value so that it always has large brackets around the parameter inside.

24 (3.18)

Why do you allow \alpha positive/negative, and have the integral from -\infty?

25 (3.20)

Use large |'s, you have the space for this, it improves readability.

25 after (3.20)

alternatively ... conveniently

26 around (3.26)

If the transition from i to j is P_{ji}, as you write, shouldn't (3.26) have a sum over i, or simply switch the indices to P_{ji}?

26 around (3.26)

Did you mean "time-independent" instead of "stationary"?

29 before (3.35)

You could say such boundary conditions are basically a reflecting boundary.

32, Definition 3.29

You should add saying that y,z is another edge.

35

What does "executed on pair with classical protocols" mean here?
Do you define or USE a standard model of quantum walk here?

36 after (4.5)

A picture of your graph would be very helpful.

Calling this a "lazy walk" is confusing, as a lazy walk is a standard term for walks that include self-loops half of the time.

37, the n=6 example

I believe this is wrong, as you don't have any a=3 steps here, which is necessary for a full graph. Moreover, what's the coin here?

45

You say you can OBSERVE that for large networks, the number of measurements decreases with a growing liveliness parameter for a large N. You do not show this in your images, you do not have two graphs with the same N and varying liveliness-parameter a. Why?

46, figure 4.3(a)

It looks to me like the initial position was around 23, not 18...
Why don't you *depict* it instead of talking about it way down on the page?

49

The intro sentence could start with: *We expect future quantum networks to consist of a large number...*

49

Is it possible to solve search problems: efficiently, or at all, or maybe with some speedup?

50

An illustration of the game would help.

51, above 5.2

Is it NP-hard for adaptive or non-adaptive Derek here?

52, point 3

Why don't you also consider measurement or fixing a particular state?

53, after Definition 5.4

It is unclear here what Magnus can do... can he call the |x> measurement? Maybe after some fixed time?

Why do you consider only probability =1 for success?

It is also unclear if they know each other's steps.

Somewhere, you should also use words to illustrate that if Magnus chooses a N/2 step, he can surely reach the opposite point.

54, the table

Are the last 2,8,4,8 perhaps extra?

55 after (5.10)

It is unclear (although you say "clearly") how attaining/visiting is done, when the |x> measurements and with which |x> are done (if at all).

57 after Proposition 5.8

Why do you have a permutation operator assumption for Magnus here?

56, 60 the figures

You should put labels on the axes.

63

In your conclusion, you evaluate Derek's best response to Magnus' classical strategy. So what? Couldn't Magnus do better?

65

In the intro sentence, you could also note quantum simulation and optimization, these are also very important.

Also, you should emphasize here what's your work, and do it after the intro, with some lead into it.

65, bottom line

due to its evaluation ... during its evaluation?

78 Def 7.4

Is the graph "labeled" by \Epsilon_{i,j}, or are the \Epsilon_{i,j}'s associated to edges?

81 Remark 7.10

It is not clear now what the "generalized" means, please, remind the reader, where you defined them.

81 intro to 7.2

Behavior of quantum walks or only open quantum walks?
You should remind the reader that MFPT stands for mean first passage time.

82 Definition 7.11

Is it a mean first passage from i to j? You should say this.
Does it also relate only to "observing" the property \Pi_v at vertex j?

82 Remark 7.2

Which N is this now, please?

83 Definition 7.15

Remind the reader that ART stands for the average return time.

83 Remark 7.16

It is misplaced to have in a remark about Apollonian networks here, if you didn't say anything about them yet. Leave this for later.

84 Figure 7.2

What is the motivation behind the choices of A,B,C, and the TOM built from them that describes the evolution here? You only say something about bistochastic operators in a cryptic remark on the end of page 85.

85 Figure 7.3

Where is the initial state in (d)? I don't see it highlighted, isn't it part of the cycle?

86 the tables

Tables are notoriously hard to read and compare. Wouldn't a graph be better? Please, at least highlight the differences!

89, table 7.3

Same remark, this is hard to compare, not even speaking about the ability to compare this to 1/d scaling. This should have been a graph!

92 Figure 7.6

What is the equation you are referencing? Same question for line 10 of page 93.

101 Figure 8.1

You should differentiate the two types of vertices by e.g. thin and thick circles.

102 Figure 8.2

You can emphasize e.g. the A vertices, and also make the arrows easier to read.

103 Figure 8.3

Say that the 2D lattice is bipartite, with 2 types of vertices.

Label your axes! Say where the walk starts! What are the boundary conditions?

Is this the large-T limit? You say this is at "various times". Where are they labeled?

There is no evolution depicted here. I expected N=something, t=1, 10, 100, ... and then another comparison for a fixed T and varying N. Now this is very hard to compare and conclude stuff.

103 above Definition 8.7

transition invariant ... translationally invariant