Discrimination of quantum measurements

Summary of doctoral dissertation

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1 English abstract

In this dissertation, we demonstrate that adaptive discrimination schemes can improve the discrimination of quantum measurements. The problem of discrimination of quantum measurements is studied in three approaches, which are: symmetric, unambiguous and asymmetric discrimination.

Symmetric discrimination is also known as minimum error discrimination. Its goal is to minimize the probability of making an erroneous decision, thus maximizing the probability that the discrimination is correct. The second approach is called unambiguous discrimination. In this approach, whenever we get the conclusive result of discrimination, we can be sure that it is correct. However, there is a chance of getting an inconclusive result of discrimination. The third approach, which is asymmetric discrimination, is also known as certification, and it is based on statistical hypothesis testing. This time, we consider the false positive and false negative errors separately.

In the basic discrimination scheme, we assume that one of two quantum measurements, which both classical descriptions are known, is secretly chosen and hidden in a black box. This box cannot be opened, but we can use measurement inside the black box. We prepare as input some quantum state, which can be entangled with some additional system. Then, we perform the measurement in the black box and basing on the measurement label, we measure the additional system. Eventually, basing on the outcome of the last measurement, we decide which of these measurements was contained in the black box.

All three approaches to the discrimination problem are first studied in the single-shot scenario, when the black box containing one of two measurements can be accessed only once. We also study the parallel discrimination scheme as well as the most general – adaptive scheme. The adaptive discrimination scheme allows for performing processing between subsequent queries to the black box, thus modifying the input for the subsequent query. The thesis of this dissertation concerns both multiple-shot discrimination schemes and states: *Adaptive schemes can improve the discrimination of quantum measurements.*

This dissertation consists of seven chapters and two appendices. The first chapter provides a general introduction and motivation. Mathematical preliminaries, basic notions of quantum information theory and various useful mathematical tools are introduced in the second chapter. Symmetric discrimination of quantum measurements is studied in single-shot and multiple-shot cases in Chapters 3 and 4, respectively. Unambiguous discrimination of quantum measurements is explored in Chapter 5. The following Chapter 6 is devoted to asymmetric discrimination. Final remarks and conclusions can be found in Chapter 7. There are also two appendices, which provide proofs which were too long and technical to be contained in the main text.

2 Polish abstract

W tej rozprawie wykazujemy, że adaptacyjne schematy rozróżniania mogą poprawić rozróżnialność pomiarów kwantowych. Istnieją trzy podstawowe podejścia do badania problemu rozróżnialności: rozróżnianie symetryczne, jednoznaczne i asymetryczne.

Celem rozróżniania symetrycznego jest zminimalizowanie prawdopodobieństwa podjęcia błędnej decyzji, a więc maksymalizacja prawdopodobieństwa, że rozróżnianie się powiodło. Drugie podejście nazywane jest rozróżnieniem jednoznacznym. Wykorzystując to podejście, jeżeli otrzymamy rozstrzygający wynik rozróżniania, możemy być pewni, że jest on poprawny. Istnieje jednak szansa uzyskania wyniku nierozstrzygającego. Trzeci podejście, jakim jest rozróżnianie asymetryczne, znane jest również jako certyfikacja i opiera się na statystycznym testowaniu hipotez. W tym podejściu rozważamy oddzielnie błędy fałszywie dodatnie i fałszywie ujemne.

W podstawowym schemacie rozróżniania zakładamy, że jeden z dwóch pomiarów kwantowych, których opisy klasyczne są nam znane, jest potajemnie wybrany i schowany w czarnej skrzynce. Tej skrzynki nie można otworzyć, ale możliwe jest użycie pomiaru, który znajduje się wewnątrz niej. Przygotowujemy jako stan wejściowy stan kwantowy, który może być splątany z dodatkowym systemem. Następnie jest on mierzony przy pomocy pomiaru znajdującego się w czarnej skrzynce. Na podstawie otrzymanej etykiety pomiaru dokonujemy pomiaru dodatkowego systemu. Jego wynik pozwala na podjęcie decyzji, który z pomiarów znajdował się w czarnej skrzynce.

Wszystkie trzy podejścia do problemu rozróżniania są w pierwszej kolejności badane w sytuacji, gdy czarna skrzynka, która zawiera jeden z dwóch pomiarów, może być dostępna tylko raz. Potem badamy również równoległy schemat rozróżniania oraz najbardziej ogólny schemat adaptacyjny, który dopuszcza wykonywanie dodatkowych procedur pomiędzy kolejnymi zapytaniami do czarnej skrzynki. Dzięki temu możliwa jest modyfikacja stanu wejściowego dla kolejnego zapytania. Teza tej rozprawy brzmi: *Schematy adaptacyjne mogą ulepszyć rozróżnianie pomiarów kwantowych.*

Niniejsza rozprawa składa się z siedmiu rozdziałów i dwóch dodatków. Rozdział pierwszy zawiera wprowadzenie i motywację, jaka towarzyszyła napisaniu tej pracy. Preliminaria matematyczne, podstawowe pojęcia kwantowej teorii informacji i wprowadzenie narzędzi matematycznych znajdują się w rozdziale drugim. Rozróżnianie symetryczne pomiarów kwantowych jest badane w sytuacjach pojedynczych i wielokrotnych odpowiednio w rozdziałach trzecim i czwartym. Badania rozróżnienia jednoznacznego znajdują się w rozdziale piątym. Rozdział szósty jest poświęcony rozróżnianiu asymetrycznemu. Końcowe uwagi i wnioski można znaleźć w rozdziale siódmym. Na końcu pracy znajdują się dwa dodatki, które zawierają dowody twierdzeń, które nie znalazły się w tekście głównym.

3 List of publications

Publications relevant to the dissertation are highlighted bold.

Published work

1. A. Glos, A. Krawiec, Z. Zimborás; *Space-efficient binary optimization for variational quantum computing*; npj Quantum Information, vol. 8, issue 1, 2022
2. **A. Krawiec, Ł. Pawela, Z. Puchała; *Excluding false negative error in certification of quantum channels*; Scientific Reports, vol. 11, pp. 21716, 2021**
3. **Z. Puchała, Ł. Pawela, A. Krawiec, R. Kukulski, M. Oszmaniec; *Multiple-shot and unambiguous discrimination of von Neumann measurements*; Quantum, vol. 5, p. 425, 2021**
4. A. Glos, A. Krawiec, Ł. Pawela; *Asymptotic entropy of the Gibbs state of complex networks*; Scientific Reports, vol. 11, no.1, pp. 1-9, 2021
5. **P. Lewandowska, A. Krawiec, R. Kukulski, Ł. Pawela, Z. Puchała; *On the optimal certification of von Neumann measurements*; Scientific Reports, vol. 19, no.1, pp. 1-16, 2021**
6. **A. Krawiec, Ł. Pawela, Z. Puchała; *Discrimination of POVMs with rank-one effects*; Quantum Information Processing, vol. 19, 2020**
7. **Z. Puchała, Ł. Pawela, A. Krawiec, R. Kukulski; *Strategies for optimal single-shot discrimination of quantum measurements*; Physical Review A, vol. 98, issue 4, 2018**
8. A. Glos, A. Krawiec, R. Kukulski, Z. Puchała; *Vertices cannot be hidden from quantum spatial search for almost all random graphs*; Quantum Information Processing, vol. 17, pp. 81, 2018
9. Z. Puchała, Ł. Rudnicki, A. Krawiec, K. Życzkowski; *Majorization uncertainty relations for mixed quantum states*; Journal of Physics A: Mathematical and Theoretical, vol. 51, issue 17, 2018

Preprints

1. A. Krawiec, Ł. Pawela, Z. Puchała; *Discrimination and certification of unknown quantum measurements*; arXiv preprint arXiv:2301.04948

4 Extended summary

Quantum technology is developing rapidly, and new quantum devices are being built every year. How to assess the quality of such new devices? Various ideas are studied [1, 2], but there is still a need for reliable theoretical tools allowing for verification of the quality of quantum devices and their components. At the end of every quantum computation is a quantum measurement, which gives classical information about the quantum system. Therefore, assessing the quality of quantum measurements is of particular importance. This dissertation addresses the problem of discrimination and certification of quantum measurements.

In the basic version of the discrimination problem, we want to distinguish between two objects. We assume that we know the descriptions of both objects, and one of these objects is secretly chosen and hidden in a black box. Unfortunately, we cannot just open the box and see what is inside. Much attention has been given to the problems of discrimination of quantum states [3–7] and channels [8–13]. Only a few works have been devoted to the problem of discrimination of quantum measurements [14–16].

Quantum channels transform one quantum state into another. When the discriminated objects are quantum channels, we can prepare an input state and perform the channel in the black box on this input state. As we can prepare any input state, we look for a state that will result in as different outputs as possible, depending on which channel was hidden in the black box. Then, we apply the channel contained in the black box to the prepared input state, and, as a result, we obtain an output state. This resulting output is also a quantum state; hence we need to measure this state to get some classical information about it. Basing on the result of the final measurement, we decide which of the two channels was hidden in the black box.

As we are considering the discrimination of quantum objects, we can take advantage of quantum entanglement to improve the discrimination. We can prepare an entangled input state on a larger space and apply the channel in the black box on only one part of the input state. Finally, we prepare a final quantum measurement and measure the resulting state. Basing on its classical output, we decide which of the channels was inside the black box.

One can consider the situation when the black box contains a quantum measurement instead of a quantum channel. We assume that we know classical descriptions of these measurements. This problem, when the black box contains a secretly chosen quantum measurement, is the focus of attention in this dissertation. The discrimination scheme begins with preparing an input state on a compound register. Then, one part of this state is measured by the measurement contained in the black box. Finally, we measure the other part of the state by a prepared quantum measurement. Basing on the outcome of this final measurement, we make a decision about which of the measurements was inside the black box.

How to assess the quality of the discrimination procedure? Quantum states can be discriminated perfectly if and only if they are orthogonal. The situation for quantum channels and measurements is more complex. In some cases, we may be able to get the correct result of discrimination with probability one, but if this is not the case, how can we assess how good the discrimination was? We may be satisfied with the situation when we know it only up to some probability. It may also happen that we want to avoid making a mistake in discrimination so much that we agree on the possibility of obtaining an inconclusive answer.

There are three basic approaches towards the discrimination problem that will be studied in this dissertation: symmetric, unambiguous, and asymmetric. Symmetric discrimination is also known as minimum error discrimination. This approach aims to decide which of the given objects was hidden in the black box with as good probability as possible. In other words, we want to minimize the probability of making a mistake. It also means that whenever we make a decision about which of the two devices was chosen, we know it only up to some probability. The second approach assumes that whenever we get a conclusive result (that is when we are able to decide which of the devices was in the black box), we know it with certainty. It may happen, however, that we are not able to make a conclusive decision. In other words, if we know which device was hidden in the black box, we know it with probability one, but there is a chance that we will get an inconclusive answer. The third approach, also known as certification, takes advantage of the statistical hypotheses testing. More precisely, we assume that one of the devices was in the black box and try to verify this hypothesis. In this approach, we differentiate two types of error: false positive and false negative errors and study them separately.

So far, we have discussed only the situation when the black box can be used only once. However, after a single query to the black box, we may not always be able to obtain satisfactory results of the discrimination. A natural solution to this problem includes using the quantum channel or measurement contained in the black box many times. The most natural extension of the single-shot scheme is the parallel discrimination scheme. One can also try to make use of extra processing between queries to the black box, which is known as the adaptive discrimination scheme. In this dissertation, we will focus on studying parallel and adaptive discrimination strategies. The thesis of this dissertation yields:

Adaptive strategies can improve discrimination of quantum measurements.

The dissertation consists of seven chapters and two appendices. The first chapter provides a general introduction and motivation. The second chapter introduces fundamental notions of quantum information theory and mathematical preliminaries.

Symmetric discrimination is studied in the third chapter in the case when the black box can be used exactly once. This chapter is based on the works [17, 18] and focuses on the symmetric discrimination of two classes of quantum measurements: projective von Neumann measurements and symmetric informationally complete measurements, which are known as SIC POVMs [19–22]. The main results of this chapter include the calculated diamond norm distance between von Neumann measurements and its geometrical interpretation. The closed-form expression of this distance allows for calculating the optimal probability of discrimination between von Neumann measurements. Bounds on the diamond norm are calculated for the distance between SIC POVMs.

Multiple-shot symmetric discrimination is studied in the fourth chapter based on the work [23]. We calculated the probability of successful discrimination between von Neumann measurements after N queries in the parallel scheme. Another result states that for the discrimination of von Neumann measurement, the parallel scheme is optimal in the sense that the use of an adaptive scheme cannot improve the probability of successful discrimination. Moreover, we presented an example of a pair of SIC POVMs of dimension three which cannot be discriminated perfectly after any finite number of queries in the parallel scheme, but can be discriminated perfectly after two queries in the adaptive scheme. The algorithm of this adaptive discrimination scheme is also contained in the dissertation.

The fifth chapter concerns unambiguous discrimination. It is based on the work [23]. The probability of unambiguous discrimination between two measurements with rank-one effects is calculated for single-shot and parallel cases. We state an expression for the probability of unambiguous discrimination of SIC POVMs and present a geometrical interpretation of this probability for discrimination of von Neumann measurements. Furthermore, we also prove that the parallel scheme is always optimal for the unambiguous discrimination of von Neumann measurements.

The asymmetric discrimination scheme is studied in the sixth chapter. It contains the results from the works [24, 25]. We proved a condition when general quantum channels could be discriminated in the asymmetric scheme; that is when we can assure that no false negative error can occur after a finite number of queries in the parallel scheme. We formulated similar conditions for general quantum measurements, von Neumann measurements, and SIC POVMs. We calculated the optimal probability of making the false positive error for SIC POVMs for single-shot and parallel schemes.

Moreover, we considered the case when we assumed an upper bound on the false positive error and wanted to find the optimal probability of making the false negative error. We calculated this optimal probability for von Neumann measurements and presented its geometrical interpretation and connection with the notion of q-numerical range [26, 27]. We also analyzed the multiple-shot discrimination and proved that the adaptive scheme could not improve the asymmetric discrimination of von Neumann measurements.

Conclusions and final remarks can be found in the seventh chapter, and technical proofs are presented in appendices.

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